SECTION : A

1

$$f(x) = 3^{x} + 3^{-x}$$

$$f'(x) = 3^{x}\log_e 3 - 3^{-x}\log_e 3 \quad \leftarrow \text{---} \; 1$$

$$= (3^{x} - 3^{-x})\log_e 3$$

જો $f'(x) = 0 \Rightarrow 3^{x} = 3^{-x}$

→ હવે જો $\forall\, x < 0$ તો $\left(\frac{1}{3}\right)^{x} > 3^{x}$

$\therefore 3^{-x} > 3^{x}$

$\therefore 3^{x} - 3^{-x} < 0$

→ જો $x > 0$ તો $3^{x} > \frac{1}{3^{x}} = 3^{-x}$ $\therefore 3^{x} - 3^{-x} > 0$

આમ $x \in (-\infty, 0)$ માટે ઘટતુ અને
$x \in (0, \infty)$ માટે વધતુ વિધેય છે. $\leftarrow \text{---} \; 1$

$\boxed{2}$

2

$$f(x) = 4x + 3\cot x$$

$$f'(x) = 4 - 3\operatorname{cosec}^2 x$$

જો $f'(x) = 0$

$\Rightarrow \operatorname{cosec}^2 x = 4/3$

$\Rightarrow \sin^2 x = 3/4$

$\Rightarrow \sin x = \pm\sqrt{3}/2 \quad \therefore x = \pi/3$ or $x = \frac{-\pi}{3} \leftarrow \text{---} \; 1$

→ હવે $f''(x) = -3(2\operatorname{cosec} x)(-\operatorname{cosec} x \cot x)$

$$= 6\operatorname{cosec}^2 x \cot x$$

SECTION - B

9 અહીં વિધેય $f(x)$ એ $[1,4]$ માં સતત છે.
$[1,4]$ નું સમાન લંબાઈના n ઉપઅંતરાલોમાં
વિભાજન કરતાં પ્રત્યેક ઉપઅંતરાલની લંબાઈ $h = \frac{b-a}{n} = \frac{3}{n}$ ---- (1)

$a = 1$, $b = 4$, $f(x) = x^3$, $nh = 3$

$$f(a+ih) = f(1+ih) = (1+ih)^3$$
$$= 1 + 3ih + 3i^2h^2 + i^3h^3 \quad \text{----} \ \tfrac{1}{2}$$

$$\therefore \int_1^4 x^3\,dx = \lim_{n\to\infty} h \sum_{i=1}^{n} f(a+ih)$$
$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \{1 + 3ih + 3i^2h^2 + i^3h^3\}$$
$$= \lim_{n\to\infty} \frac{3}{n} \left\{ \Sigma 1 + 3h\Sigma i + 3h^2 \Sigma i^2 + h^3 \Sigma i^3 \right\} \quad \text{----} \ \tfrac{1}{2}$$

$$= \lim_{n\to\infty} \frac{3}{n} \left\{ n + 3\left(\frac{3}{n}\right)\cdot\frac{n(n+1)}{2} + 3\left(\frac{9}{n^2}\right)\frac{n(n+1)(2n+1)}{6} + \frac{27}{n^3}\cdot\frac{n^2(n+1)^2}{4} \right\}$$

$$= \lim_{n\to\infty} 3 \left\{ 1 + \frac{9}{2}\left(1+\frac{1}{n}\right) + \frac{27}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + \frac{27}{4}\left(1+\frac{1}{n}\right)^2 \right\} \quad \tfrac{1}{2}$$

$$= 3 \left\{ \frac{27}{4}(1+0)^2 + \frac{9}{2}(1+0)(2+0) + \frac{9}{2}(1+0) + 1 \right\}$$

$$= 3 \left\{ \frac{27}{4} + 9 + \frac{9}{2} + 1 \right\}$$

$$= 3 \left(\frac{27+36+18+4}{4} \right) = 3 \times \frac{85}{4} = \frac{255}{4} \quad \text{----} \ \tfrac{1}{2}$$

$=$

$\boxed{3}$

$$\therefore 2I = \pi \int_{-1}^{1} \frac{1}{1+t^2}\, dt$$

$$I = \frac{\pi}{2} \left[\tan^{-1} t \right]_{-1}^{1}$$

$$= \frac{\pi}{2} \left[\tan^{-1}(1) - (\tan^{-1}(-1)) \right]$$

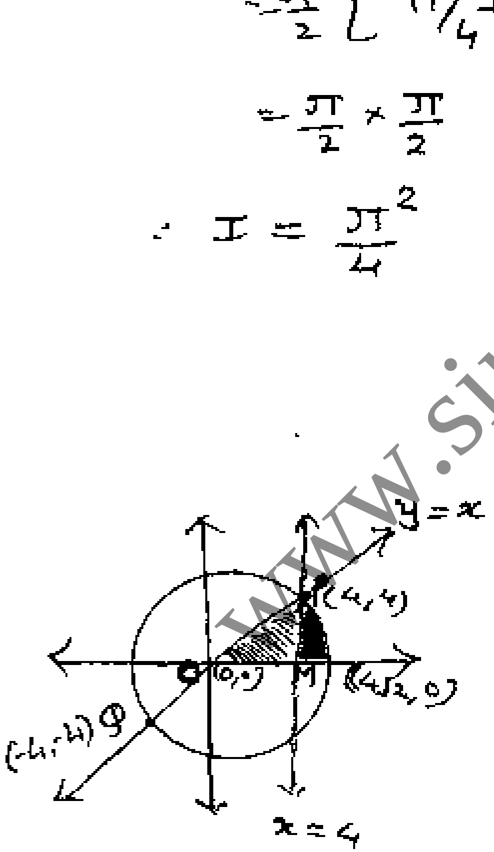
$$= \frac{\pi}{2} \left[\pi/4 + \frac{\pi}{4} \right] = \frac{2\pi}{4}$$

$$= \frac{\pi}{2} \times \frac{\pi}{2} = \frac{\pi^2}{4}$$

$$\therefore I = \frac{\pi^2}{4} \text{ ----- } \frac{1}{2}$$

$\boxed{3}$

$\boxed{10}$



અહીં $x^2+y^2=32$ વર્તુળ છે

કેન્દ્ર $(0,0)$, ત્રિજ્યા $r=4\sqrt{2}$

હવે રેખા $y=x$ અને

$x^2+y^2=32$ ના છેદબિંદુ માટે

$2x^2=32 \quad \therefore x^2=16$

$\therefore x=\pm 4 \quad \text{----- } \frac{1}{2}$

પ્રથમ ચરણમાં આવૃત્ત માટે $x>0$ તેથી $x=4$

$\therefore y=x=4 \quad \therefore$

$\therefore$ છેદબિંદુઓ $P(4,4)$, $Q(-4,-4)$

અહીં $x^2+y^2=32$ અને $x=y$ દ્વારા આવૃત્ત પ્રદેશ પ્રથમ ચરણમાં વિચારતા

$A = |I_1| + |I_2|$ ← - - - - - $\frac{1}{2}$

આકૃતિ મુજબ $|I_1|$ = Δ OMP નું ક્ષેત્રફળ

$|I_2|$ = $\widehat{AP}$ ની અંદરના ભાગમાં વર્તુળનો પ્રથમ ચરણમાં ભાગ

$\therefore y^2 = 32 - x^2$

$y = \pm\sqrt{32-x^2}$

$y = \sqrt{32-x^2} \quad (\because y > 0)$

હવે $I_1 = \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = 8$ ← —— $\frac{1}{2}$

(અથવા $\Delta = \frac{1}{2}$ આધાર × ઊંચાઈ)

$I_2 = \int_4^{4\sqrt{2}} \sqrt{32-x^2}\,dx$

$= \left[\frac{x}{2}\sqrt{32-x^2} + \frac{32}{2}\sin^{-1}\frac{x}{4\sqrt{2}}\right]_4^{4\sqrt{2}}$

$= \left[16\cdot\frac{\pi}{2} - \left(2\cdot 4 + 16\left(\frac{\pi}{4}\right)\right)\right]$

$= 8\pi - 8 - 4\pi$ ← - - - - - - - 1

$I_2 = 4\pi - 8$

$\therefore A = |I_1| + |I_2| = 8 + 4\pi - 8 = 4\pi$ ← —— $\frac{1}{2}$

$\boxed{3}$

$\therefore \boxed{A = 4\pi}$

$\boxed{11}$ $(1+e^{x/y})dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

રસ્ત·

$$\therefore \frac{dx}{dy} = \frac{e^{x/y}\left(x/y - 1\right)}{1+e^{x/y}} = \phi\left(\frac{x}{y}\right) \leftarrow \text{---} \frac{1}{2}$$

— (1)

જે સમપરિમાણીય વિકલ સમીકરણ છે.

ધારો કે $\frac{x}{y} = v \quad \therefore x = vy$

$$\therefore \frac{dx}{dy} = v + y\frac{dv}{dy} \quad \text{— (2)} \leftarrow \text{-----} \frac{1}{2}$$

(1) અને (2) પરથી,

$$v + y\frac{dv}{dy} = \frac{e^{v}(v-1)}{1+e^{v}}$$

$$\therefore y\frac{dv}{dy} = \frac{ve^{v} - e^{v} - v - e^{v}\cdot v}{1+e^{v}}$$

$$\therefore y\frac{dv}{dy} = -\frac{v+e^{v}}{1+e^{v}}$$

$$\therefore \frac{1+e^{v}}{v+e^{v}}dv + \frac{1}{y}dy = 0 \quad \leftarrow \text{-----} 1$$

જે વિયોજનીય ચલ સ્વરૂપનું વિકલ સમીકરણ છે.

$$\therefore \int \frac{1+e^{v}}{v+e^{v}}dv + \int \frac{1}{y}dy = \log|c|$$

$$\therefore \log|v+e^{v}| + \log|y| = \log|c|$$

$$\therefore \log|(v+e^{v})\cdot y| = \log|c|$$

$$\therefore \left(\frac{x}{y} + e^{x/y}\right)y = c$$

$$\therefore x + ye^{x/y} = c \quad \leftarrow \text{-------} 1$$

$\boxed{3}$

$\boxed{11}$ આપેલ $\dfrac{dy}{dx} + \dfrac{4x}{x^2+1}y = \dfrac{1}{(x^2+1)^3}$

$P(x) = \dfrac{4x}{x^2+1} \qquad Q(x) = \dfrac{1}{(x^2+1)^3}$ ← $\frac{1}{2}$

$\therefore e^{\int P(x)dx} = e^{2\int \frac{2x}{x^2+1}dx}$

$= e^{2\log_e(x^2+1)}$

$= (x^2+1)^2$ ← $\frac{1}{2}$

$(x^2+1)^2\dfrac{dy}{dx} + 4x(x^2+1)y = \dfrac{1}{x^2+1}$

$\dfrac{d}{dx}\left(y\cdot(x^2+1)^2\right) = \dfrac{1}{x^2+1}$ ← 1

$\therefore y(x^2+1)^2 = \int \dfrac{1}{x^2+1}dx$

$\therefore y(x^2+1)^2 = \tan^{-1}x + c$ ← 1

જ્યાં c સ્વૈર અચળ

$\boxed{3}$

આમ $y(1+x^2)^2 = \tan^{-1}x + c$ એ આપેલા વિકલ સમીકરણનો વ્યાપક ઉકેલ છે.

$$\therefore f''(\pi/3) = 6\left(\operatorname{cosec}\pi/3\right)^2 \cot \pi/3$$
$$= 6 \times \frac{4}{3} \times \frac{1}{\sqrt{3}} = +\frac{8}{\sqrt{3}} > 0$$
$$f''(-\pi/3) = 6\left(\operatorname{cosec}(-\pi/3)\right)^2 \cot(-\pi/3)$$
$$= 6 \times \frac{4}{3} \times -\frac{1}{\sqrt{3}} = -\frac{8}{\sqrt{3}} < 0$$

$x = \pi/3$ આગળ f ને સ્થાનીય ન્યૂનતમ મૂલ્ય અને $x = -\pi/3$ આગળ સ્થાનીય મહત્તમ મૂલ્ય મળે.

$\therefore$ સ્થાનીય ન્યૂનતમ મૂલ્ય $f\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + 3\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{4\pi}{3} + \sqrt{3}$ ←--- $\frac{1}{2}$

સ્થાનીય મહત્તમ મૂલ્ય $f\left(-\frac{\pi}{3}\right) = -\frac{4\pi}{3} + 3\left(\frac{-1}{\sqrt{3}}\right)$

$= -\frac{4\pi}{3} - \sqrt{3}$ ←--- $\frac{1}{2}$

સ્થાનીય ન્યૂનતમ મૂલ્ય $= \frac{4\pi}{3} + \sqrt{3}$ $\boxed{2}$

સ્થાનીય મહત્તમ મૂલ્ય $= -\frac{4\pi}{3} - \sqrt{3}$

$\boxed{3}$ $I = \int x^7 \sin x^4 \, dx$

ધારો કે $x^4 = t$

$4 \cdot x^3 \, dx = dt$

$\therefore x^3 \, dx = \frac{1}{4} dt$ ←---- $\frac{1}{2}$

$$\therefore I = \int x^4 \sin x^4 \cdot x^3\,dx$$

$$= \frac{1}{4}\int t \sin t\,dt \quad \leftarrow \frac{1}{2}$$

$$= \frac{1}{4}\left\{ t\int \sin t\,dt - \int\left(\frac{d}{dt}(t)\int \sin t\,dt\right)dt \right\}$$

$$= \frac{1}{4}\left\{ t(-\cos t) - \int(-\cos t)\,dt \right.$$

$$= -\frac{1}{4} t\cos t + \frac{1}{4}\sin t + c \quad \leftarrow \frac{1}{2}$$

$$= -\frac{x^4}{4}\cos x^4 + \frac{1}{4}\sin x^4 + c \quad \leftarrow \frac{1}{2}$$

$\boxed{2}$

$\boxed{4}$ $$I = \int \frac{x^2 - x + 1}{(x+1)^3}\,dx$$

ધારો કે $x + 1 = t \quad \therefore dx = dt \quad \leftarrow \frac{1}{2}$

$\therefore x = t - 1$

$$\therefore I = \int \frac{(t-1)^2 - (t-1) + 1}{t^3}\,dt$$

$$= \int \frac{t^2 - 2t + 1 - t + 1 + 1}{t^3}\,dt \quad \leftarrow \frac{1}{2}$$

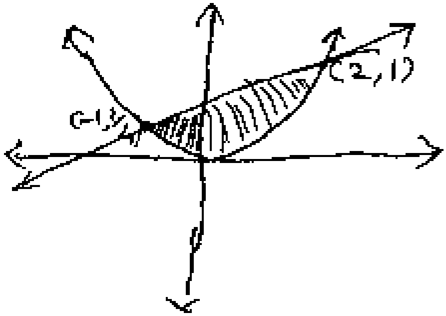
$$= \int \frac{1}{t}\,dt - 3\int t^{-2}\,dt + 3\int t^{-3}\,dt$$

$$= \log|t| + \frac{3}{t} - \frac{3}{2t^2} + c$$

$$= \log|x+1| + \frac{3}{x+1} - \frac{3}{2(x+1)^2} + c \quad \leftarrow \frac{1}{2}$$

$\boxed{2}$

$\boxed{5}$



અહીં $x^2 = 4y$, $x = 4y-2$

$x^2 = 4\left(\frac{x+2}{4}\right)$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$, $\cancel{x} = -1$

$y = 1$ $\quad y = 1/4$

$\therefore$ છેદબિંદુ $(2,1)$ અને $(-1, 1/4)$ $\leftarrow \frac{1}{2}$

હવે $\quad I = \int_a^b (f_1(x) - f_2(x))\,dx$

$= \int_{-1}^{2} \left(\frac{x+2}{4} - \frac{x^2}{4}\right) dx$

$= \frac{1}{4}\int_{-1}^{2} (x+2-x^2)\,dx \leftarrow \cdots \frac{1}{2}$

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{4}\left\{\frac{10}{3} - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)\right\}$

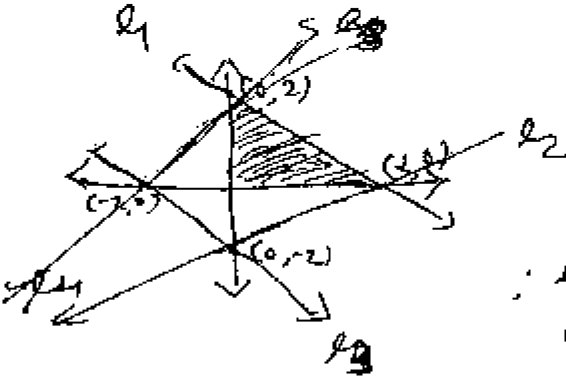
$= \frac{1}{4}\left\{\frac{10}{3} - \frac{3-12+2}{6}\right\}$

$= \frac{1}{4}\left\{\frac{10}{3} + \frac{7}{6}\right\}$

$= \frac{1}{4} \times \frac{27}{6} = \frac{1}{4} \times \frac{9}{2} = \frac{9}{8}$ $\quad \therefore A = |I| \cdots 1$

$= 9/8$ $\quad \boxed{2}$

$\boxed{5}$ આકૃતિ



$l_1 : x + y - 2 = 0$

$l_2 : x - y - 2 = 0$

$l_3 : x + y + 2 = 0$

$l_4 : x - y + 2 = 0$

$\therefore l_1 \cap l_2 = \{(2,0)\}$

$l_2 \cap l_3 = \{(0,-2)\}$

$l_3 \cap l_4 = \{(-2,0)\}$

$l_4 \cap l_1 = \{(0,2)\}$ ← - - - 1

આકૃતિ બંને અક્ષો પ્રત્યે સંમિત છે.

$\therefore A = 4(I)$ જ્યાં $I = \int_0^2 (2-x)\,dx$

$= \left[2x - \frac{x^2}{2}\right]_0^2$

$= 2$

$A = 4I = 2 \times 4 = 8$

← - - - 1

$\therefore$ ans: $\boxed{A = 8}$

$\boxed{2}$

(6) જો સદિશ $\bar{x}$ એ X અક્ષ, Y અક્ષ અને Z અક્ષ સાથે અનુક્રમે α, β અને γ માપના ખૂણા બનાવે તો

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \leftarrow \frac{1}{2}$$

અહીં $\beta = \pi/3$, $\gamma = \frac{2\pi}{3}$

$$\therefore \cos^2\alpha + \cos^2\frac{\pi}{3} + \cos^2\frac{2\pi}{3} = 1 \leftarrow \frac{1}{2}$$

$$\therefore \cos^2\alpha + \frac{1}{4} + \frac{1}{4} = 1$$

$$\therefore \cos^2\alpha = \frac{1}{2} \qquad \cos\alpha = \pm\frac{1}{\sqrt{2}}$$

$\therefore \alpha = \frac{\pi}{4}$ અથવા $\alpha = \frac{3\pi}{4} \leftarrow \frac{1}{2}$ $\boxed{2}$

(7) ધારો કે $\bar{x} = (x_1, x_2, x_3)$ માંગેલ સદિશ છે

$$\therefore \sqrt{x_1^2 + x_2^2 + x_3^2} = 1$$

$$\therefore x_1^2 + x_2^2 + x_3^2 = 1 \quad \text{—(1)}$$

હવે $(\bar{x}, \hat{i}) = \pi/4$

$$\cos\frac{\pi}{4} = \frac{\bar{x}\cdot\hat{i}}{|\bar{x}||\hat{i}|} = x_1 \qquad \therefore x_1 = \frac{1}{\sqrt{2}} \leftarrow \frac{1}{2}$$

હવે $\bar{x} \perp \hat{k}$ $\therefore (x_1, x_2, x_3)\cdot(0,0,1) = 0 \leftarrow \frac{1}{2}$

$$\therefore x_3 = 0$$

હવે $x_1^2+x_2^2+x_3^2=1$

$\therefore \frac{1}{2}+0+x_2^2=1$

$\therefore x_2^2=\frac{1}{2} \quad \therefore x_2=\pm\frac{1}{\sqrt{2}}$ ← ------ $\frac{1}{2}$

માંગેલ સદિશો $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}},0\right),\left(\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0\right)$ ← -- $\frac{1}{2}$

$\boxed{2}$

$\boxed{8}$ $\bar{l}=\left(1,\frac{1}{2},-\frac{1}{12}\right)=(12,6,-1)$

$\bar{m}=\left(2,2,\frac{1}{3}\right)=(6,6,1)$

$\bar{l}\times\bar{m}=\left(\begin{vmatrix}6&-1\\6&1\end{vmatrix},-\begin{vmatrix}12&-1\\6&1\end{vmatrix},\begin{vmatrix}12&6\\6&6\end{vmatrix}\right)$

$=(12,-18,36)$

$=6(2,-3,6)$ ← -------- $\frac{1}{2}$

માંગેલ રેખાનું સમી. $\bar{r}=\bar{a}+k(\bar{l}\times\bar{m})$, $k\in R$ ← --- $\frac{1}{2}$

$\bar{r}=(2,-1,2)+k(2,-3,6)$

અથવા, $\bar{r}=(2,-1,2)+k(12,-18,36)$

પણ લખી શકાય

આમ, → $\bar{r}=(2,-1,2)+k(12,-18,36)$ ← ---- 1

OR - અથવા

OR → $=(2,-1,2)+k(2,-3,6),\ k\in R$

$\boxed{2}$

OR - અથવા

OR → $\bar{r}=(2,-1,2)+k\left(\frac{1}{3},-\frac{1}{2},1\right);\ k\in R$.

[8] ઉકેલ

અહીં L: $\bar{r} = (1,4,-1) + k(2,-3,3),\ k \in R$

જો બિંદુ $(2,0,1) \in L$ તો

$(2,0,1) = (1+2k,\ 4-3k,\ -1+3k)$; કોઈક $k \in R$

$\therefore 1+2k = 2,\quad 4-3k = 0,\quad 1 = -1+3k$

$\therefore k = 1/2,\quad k = 4/3,\quad k = 2/3$

જે શક્ય નથી.

$\therefore (2,0,1) \notin L$ ←------- $\frac{1}{2}$

$A(\bar{a}) = (1,4,-1),\ B(\bar{b}) = (2,0,1)$

$\overrightarrow{AB} = (1,-4,2),\ \bar{l} = (2,-3,3)$

$\therefore \bar{n} = \overrightarrow{AB} \times \bar{l} = (1,-4,2) \times (2,-3,3)$

$\therefore \bar{n} = \left(\begin{vmatrix} -4 & 2 \\ -3 & 3 \end{vmatrix},\ -\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix},\ \begin{vmatrix} 1 & -4 \\ 2 & -3 \end{vmatrix} \right)$

$= (-12+6,\ -(3-4),\ -3+8)$

$= (-6,\ +1,\ 5)$ ←------- $\frac{1}{2}$

$\therefore (x,y,z)\cdot(-6,1,5) = (1,4,-1)\cdot(-6,1,5)$

$-6x+y+5z = -6+4-5$

$-6x+y+5z = -7$

$\therefore 6x-y-5z-7 = 0$ ←------- 1

$\boxed{2}$

SECTION - B

$\boxed{9}$ અહીં વિધેય $f(x)$ એ $[1,4]$ માં સતત છે.
$[1,4]$ નું સમાન લંબાઈના n ઉપઅંતરાલોમાં
વિભાજન કરતાં પ્રત્યેક ઉપઅંતરાલની લંબાઈ $h = \frac{b-a}{n} = \frac{3}{n}$ ---- (1)

$a = 1$, $b = 4$, $f(x) = x^3$, $nh = 3$

$$f(a+ih) = f(1+ih) = (1+ih)^3$$
$$= 1 + 3ih + 3i^2h^2 + i^3h^3 \quad \text{----} \ \tfrac{1}{2}$$

$$\therefore \int_1^4 x^3\,dx = \lim_{n\to\infty} h \sum_{i=1}^{n} f(a+ih)$$
$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \{1 + 3ih + 3i^2h^2 + i^3h^3\}$$
$$= \lim_{n\to\infty} \frac{3}{n} \left\{ \Sigma 1 + 3h \Sigma i + 3h^2 \Sigma i^2 + h^3 \Sigma i^3 \right\} \quad \text{----} \ \tfrac{1}{2}$$

$$= \lim_{n\to\infty} \frac{3}{n} \left\{ n + 3\left(\frac{3}{n}\right)\cdot\frac{n(n+1)}{2} + 3\left(\frac{9}{n^2}\right)\frac{n(n+1)(2n+1)}{6} + \frac{27}{n^3}\cdot\frac{n^2(n+1)^2}{4} \right\}$$

$$= \lim_{n\to\infty} 3 \left\{ 1 + \frac{9}{2}\left(1+\frac{1}{n}\right) + \frac{27}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right) + \frac{27}{4}\left(1+\frac{1}{n}\right)^2 \right\} \quad \text{----} \ \tfrac{1}{2}$$

$$= 3 \left\{ \frac{27}{4}(1+0)^2 + \frac{9}{2}(1+0)(2+0) + \frac{9}{2}(1+0) + 1 \right\}$$

$$= 3 \left\{ \frac{27}{4} + 9 + \frac{9}{2} + 1 \right\}$$

$$= 3\left(\frac{27+36+18+4}{4}\right) = 3 \times \frac{85}{4} = \frac{255}{4} \quad \text{----} \ \tfrac{1}{2}$$

$=$

$\boxed{3}$

(12) $\bar{x} \perp \bar{y}$ અને $\bar{x} \perp \bar{z}$

$|\bar{x}| = |\bar{y}| = |\bar{z}| = 1$

$\therefore$ $\bar{x}$ અને $\bar{y} \times \bar{z}$ [illegible] સમાંતર હશે.

$\therefore \bar{x} = k(\bar{y} \times \bar{z}) \quad k \in R - \{0\}$ ← $\frac{1}{2}$

હવે, $(\bar{y} \,\hat{,}\, \bar{z}) = \pi/6$

$\therefore \sin \pi/6 = \dfrac{|\bar{y} \times \bar{z}|}{|\bar{y}||\bar{z}|}$ ← $\frac{1}{2}$

$\frac{1}{2} = |\bar{y} \times \bar{z}|$

$\therefore 1 = |2(\bar{y} \times \bar{z})|$

$\therefore |\bar{x}| = |2(\bar{y} \times \bar{z})|$ ← 1

$\therefore |\bar{x}| = |\pm 2(\bar{y} \times \bar{z})|$ —— (1)

હવે $\bar{x} \times (\pm 2(\bar{y} \times \bar{z})) = \pm 2(\bar{x} \times (\bar{y} \times \bar{z}))$

$= \pm 2\{(\bar{x} \cdot \bar{z})\bar{y} - (\bar{x} \cdot \bar{y})\bar{z}\}$

$= 0$

$\bar{x}$ અને $\pm 2(\bar{y} \times \bar{z})$ સમરેખ છે. ← 1

$\therefore \bar{x} = \pm 2(\bar{y} \times \bar{z})$ —— (2)

(1) અને (2) પરથી $\bar{x} = \pm 2(\bar{y} \times \bar{z})$ $\boxed{3}$

(13) $L_1 : \{(1+3k, -1+2k, 1+5k) \mid k \in R\}$

$L_2 : \{(-2+4t, 1+3t, -1-2t) \mid t \in R\}$

$\bar{a} = (1,-1,1)$, $\bar{l} = (3,2,5)$

$\bar{b} = (-2,1,-1)$, $\bar{m} = (4,3,-2)$ ← $\frac{1}{2}$

$\therefore \bar{a}-\bar{b} = (3,-2,2)$, $\bar{l}\times\bar{m} = (-19,26,1) \neq \bar{0}$ ← $\frac{1}{2}$

$\therefore (\bar{a}-\bar{b})\cdot(\bar{l}\times\bar{m}) = (3,-2,2)\cdot(-19,26,1)$

$= -57-52+2$ ← 1

$= -107 \neq 0$

∴ આપેલી રેખાઓ વિષમતલીય છે.

∴ લઘુત્તમ અંતર $= \dfrac{|(\bar{a}-\bar{b})\cdot(\bar{l}\times\bar{m})|}{|\bar{l}\times\bar{m}|} = \dfrac{107}{\sqrt{361+676+1}}$ ← 1

$= \dfrac{107}{\sqrt{1038}}$ $\boxed{3}$

$\boxed{24}$ $\pi_1 : 2x+3y-z-4=0$

$\pi_2 : x+y+z-2=0$

અહીં બિંદુ $(1,2,2)$ માટે $2(1)+3(2)-2-4=2\neq 0$

અને $(1,2,2)$ માટે $1+2+2-2=3\neq 0$

$\therefore (1,2,2)\notin\pi_2$ ← $\frac{1}{2}$

$\therefore (1,2,2)\notin\pi_1$ અને $(1,2,2)\notin\pi_2$

∴ બંને સમતલ એ માંગેલ સમતલ નથી

ધારો કે માંગેલ સમતલનું સમીકરણ

$(2x+3y-z-4)+\lambda(x+y+z-2)=0$ ← $\frac{1}{2}$

$\therefore (2+\lambda)x+(3+\lambda)y+(-1+\lambda)z-4-2\lambda=0$

વળી તે (1, 2, 2) માંથી પસાર થાય છે.

$\therefore 2+\lambda+6+2\lambda-2+2\lambda-4-2\lambda=0$

$3\lambda=-2 \quad \therefore \lambda=-2/3 \quad \leftarrow \cdots \frac{1}{2}$

∴ માંગેલ સમતલનું સમીકરણ,

$(2x+3y-z-4)-\frac{2}{3}(x+y+z-2)=0$

$\therefore 6x+9y-3z-12-2x-2y-2z+4=0$

$\therefore 4x+7y-5z-8=0 \quad \leftarrow \cdots \frac{1}{2}$

હવે છેદરેખાની દિશા $\bar{l}$ હોય તો,

$\bar{l}=\bar{n}_1 \times \bar{n}_2$

$=(2,3,-1)\times(1,1,1)$

$=(4,-3,-1) \quad \leftarrow \cdots \frac{1}{2}$

રેખા પરના બિંદુ માટે બંને સમતલના સમીકરણમાં $z=0$ લઈએ તો,

$2x+3y-4=0 \quad \therefore y=0$

$x+y-2=0 \quad x=2$

∴ બિંદુ $(2,0,0)$ મળે.

∴ છેદરેખાનું સમીકરણ $\bar{r}=(2,0,0)+k(4,-3,-1)$, $k\in R$ $\leftarrow \cdots \frac{1}{2}$

<u>અથવા</u> $\frac{x-2}{4}=\frac{y}{-3}=\frac{z}{-1}$

$\boxed{3}$

SECTION-C

15 આપેલ વક્ર $y = lx^3 + mx^2 + nx + 5$ આગળ સ્પર્શે છે
જે X અક્ષને Q $(-2, 0)$

$\therefore -8l + 4m - 2n + 5 = 0$ —— (1)   1

જે Y અક્ષને P આગળ છેદે છે

ધારો કે P $(0, y_0)$ છે.

હવે P આગળ સ્પર્શકનો ઢાળ 3 છે.   1

$\therefore \left(\frac{dy}{dx}\right)_{(0, y_0)} = 3$

$\therefore 3lx^2 + 2mx + n = 3$

હવે $x = 0$ મૂકતાં $\boxed{n = 3}$

હવે $\left(\frac{dy}{dx}\right)_{(-2,0)} = 0$

$3lx^2 + 2mx + n = 0$

$\therefore 12l - 4m + n = 0$ —— (3)   1

$n = 3$ મૂકતાં
$$\begin{aligned} 12l - 4m + 3 &= 0 \\ -8l + 4m - 1 &= 0 \\ \hline 4l &= -2 \end{aligned}$$

$\therefore \boxed{l = -\frac{1}{2}}$

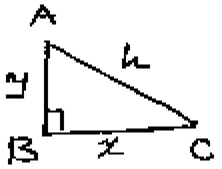
(3) પરથી $12(-\frac{1}{2}) - 4m + 3 = 0 \quad -4m = 3$

$\therefore \boxed{m = -3/4}$

$\boxed{l = -\frac{1}{2}, \ m = -3/4, \ n = 3}$ 1

$\boxed{15}$ ઉકેલ:

ધારો કે કાટકોણ ΔABC માં



$BC = x,\ AB = y$

$AC = h$ તથા $m\angle ABC = \frac{\pi}{2}$

$\therefore \Delta ABC$ નું ક્ષેત્રફળ $= \frac{1}{2} \cdot BC \times AB =$

$= \frac{1}{2} xy$

1

$\therefore \Delta = \frac{1}{2} x \cdot \sqrt{h^2 - x^2}$

$\Delta^2 = \frac{x^2}{4}(h^2 - x^2) \leftarrow \text{----} 1$

ધારો કે $f(x) = x^2(h^2 - x^2) = h^2x^2 - x^4$

હવે Δ મહત્તમ બને તે માટે

$f'(x) = 0 \Rightarrow 2h^2x - 4x^3 = 0 \leftarrow \text{---} 1$

$\Rightarrow 2x(x^2 + y^2) - 4x^3 = 0$

$\Rightarrow x[2(x^2 + y^2) - 4x^2] = 0$

$\Rightarrow 2x(y^2 - x^2) = 0$

$\Rightarrow x = 0$ or $x = y \leftarrow \text{--} 1$

પરંતુ $x \neq 0 \quad \therefore x = y$

1

હવે $f''(x) = 2h^2 - 12x^2$

$= 2(x^2 + y^2) - 12x^2$

$= 2y^2 - 10x^2 = 2(x^2) - 10x^2 = -8x^2$

$\therefore f''(x) = -8x^2 < 0 \leftarrow \text{---} 1$

$\therefore y = x$ આગળ મહત્તમ બને

$\therefore BC = AB = x = y$

$\therefore$ ત્રિકોણ સમદ્વિબાજુ થશે. $\leftarrow \text{----} 1$

4

36

$$I = \int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\,dx$$

$$= \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx + \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

$I = I_1 + I_2$ જ્યાં $I_1 = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx$

$I_2 = \int_{-\pi}^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$ ← $\frac{1}{2}$

હવે ધારો કે $f(x) = \frac{2x}{1+\cos^2 x}$,

$g(x) = \frac{2x\sin x}{1+\cos^2 x}$

$\rightarrow \because f(-x) = \frac{2(-x)}{1+\cos^2(-x)} = \frac{-2x}{1+\cos^2 x} = -f(x)$ ← $\frac{1}{2}$

$\therefore$ f એ અયુગ્મ વિધેય છે.

$\therefore I_1 = \int_{-\pi}^{\pi} \frac{2x}{1+\cos^2 x}\,dx = 0$

$\rightarrow g(-x) = \frac{2(-x)\sin(-x)}{1+\cos^2(-x)} = \frac{2x\sin x}{1+\cos^2 x} = g(x)$

$\therefore$ g(x) યુગ્મ વિધેય છે. ← $\frac{1}{2}$

$$\therefore I_2 = 2\int_0^{\pi} \frac{2x\sin x}{1+\cos^2 x}\,dx$$

$$\therefore I_2 = 4\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$= 4\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx$$

$$= 4\int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\,dx$$

$$= 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx - 4\int \frac{x\sin x}{1+\cos^2 x}\,dx$$

$$\therefore I_2 = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx - I_2$$

$$\therefore 2I_2 = 4\pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx \leftarrow 1$$

ધારો કે $\cos x = t$ $\qquad \sin x\,dx = -dt$

અને $x=0 \Rightarrow t=1$ $\qquad \leftarrow \frac{1}{2}$

$x=\pi \Rightarrow t=-1$

$$\therefore 2I_2 = 4\pi\int_{-1}^{1} \frac{-dt}{1+t^2}$$

$$\therefore I_2 = 2\pi \int_{-1}^{1} \frac{1}{t^2+(1)^2}\, dt$$

$$= 2\pi \left[\tan^{-1} t\right]_{-1}^{1}$$

$$= 2\pi \left(\frac{\pi}{4} + \frac{\pi}{4}\right)$$

$$= 2\pi \left(\frac{\pi}{2}\right) = \pi^2$$

$$\therefore I = I_1 + I_2 = 0 + \pi^2 = \pi^2$$

$$\therefore \boxed{I = \pi^2} \quad \leftarrow 1$$

$\boxed{4}$

$\boxed{17}$ $$I = \int \frac{x^2+x+1}{(x+1)^2(x+2)}\, dx$$

ધારો $$\frac{x^2+x+1}{(x+1)^2(x+2)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+2} \leftarrow \frac{1}{2}$$

$$\therefore x^2+x+1 = A(x+1)(x+2) + B(x+2) + C(x+1)^2$$

$\rightarrow x=-1$ લેતાં, $1 = B(-1+2)$ $\quad \therefore B = 1$

$\rightarrow x=-2$ લેતાં $3 = C(-2+1)^2$ $\quad \therefore C = 3$ $\Big\} \leftarrow 1$

$\rightarrow x=0$ લેતાં $1 = A(1)(2) + B(2) + C(1)$

$$\therefore 2A + 2B + C = 1$$

$$2A + 2(1) + 3 = 1$$

$$A = -2 \quad \leftarrow \frac{1}{2}$$

$\therefore A = -2, \; B = 1, \; C = 3$

$$I = \int \left(\frac{-2}{x+1} + \frac{1}{(x+1)^2} + \frac{3}{x+2} \right) dx \leftarrow \frac{1}{2}$$

$$= -2\int \frac{1}{x+1}\, dx + \int (x+1)^{-2}\, dx + 3\int \frac{1}{x+2}\, dx \leftarrow \frac{1}{2}$$

$$= -2 \log|x+1| - \frac{1}{x+1} + 3 \log|x+2| + c$$

$$= 3 \log|x+2| - 2 \log|x+1| - \frac{1}{x+1} + c \leftarrow 1$$

$\boxed{4}$

18 જો S એ આસપાસના વાતાવરણનું અચળ તાપમાન હોય તો,

$$\frac{dT}{dt} \propto (T-S)$$

$\therefore \frac{dT}{dt} = -K(T-S)$ (∵ તાપમાન ઘટે છે) ← 1

$$\therefore \frac{dT}{T-S} = -K\,dt$$

$$\therefore \int \frac{1}{T-S}\,dt = -K\int 1\,dt$$

$\therefore \log(T-S) = -Kt + c$ — (1)

→ જ્યારે $t=0$ ત્યારે $T=80°F$ ← 1

$$\therefore \log(80-S) = c$$

(1) પરથી, $\log(T-S) = -Kt + \log(80-S)$

→ જ્યારે $t=5$ ત્યારે $T=60°F$

$\therefore \log(60-S) = -5K + \log(80-S)$ — (2)

→ જ્યારે $t=10$ ત્યારે $T=50°F$

$\therefore \log(50-S) = -10K + \log(80-S)$ — (3) ← $\frac{1}{2}$

(1) (2) અને (3) પરથી

$$\frac{1}{5}\log\left(\frac{60-S}{80-S}\right) = -K = \frac{1}{10}\log\left(\frac{50-S}{80-S}\right)$$

$$\therefore 2\log\left(\frac{60-S}{80-S}\right) = \log\left(\frac{50-S}{80-S}\right)$$

$\therefore \left(\frac{60-S}{80-S}\right)^2 = \frac{50-S}{80-S}$ ← $\frac{1}{2}$

$\therefore (60-S)^2 = (80-S)(50-S)$

$\therefore 3600 - 120S + S^2 = 4000 - 130S + S^2$

$10S = 400 \quad \therefore S = 40$

ઓરડાનું તાપમાન 40°F છે. 4 - - 1

$\boxed{4}$